A Reduced-Gravity Isopycnal Ocean Model: Hindcasts of El Niño

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ABSTRACT

A global isopycnal ocean model is presented for the study of interannual to interdecadal variability in the global ocean. The model treats the primitive equations on a sphere with a generalized vertical coordinate. This coordinate is designed to represent a turbulent well-mixed surface layer and nearly isopycnal deeper layers. Disappearing isopycnals are treated through the quasi-isopycnal technique, in which the coordinate separates from the isopycnal in order to maintain a minimum layer thickness. A reduced gravity treatment is made, with the deepest interface at a mean depth of 2300 m. Coastal topography is represented, but the reduced gravity treatment precludes the use of variable bottom depth. The model is used for hindcast studies of El Niño during the decade from 1982 through 1991 using a combination of climatological wind forcing and wind anomalies derived from various sources. In order to carry out the hindcast experiments, a technique is developed for constructing a mean climatological surface heat flux using the model, climatological wind forcing, and climatological surface temperatures. In the hindcast runs, the climatological winds and heat flux are augmented by the wind anomalies and a weak damping of surface temperature anomalies. A series of tests compares different data products for the wind anomalies. The first product is obtained from the Florida State University (FSU) wind analysis. The second and third wind products are obtained from global climate GCM simulations run over observed sea surface temperatures (SST). Although the wind products appear quite similar, the model results show large differences in hindcast skill, reflecting the fact that subtle features of the winds can have large impacts on ocean simulations and can be seen as a primary cause of wide differences in coupled GCM performance. The model maintains a sharp thermocline and a strong equatorial undercurrent in the center of the ocean basin. The heat flux needed to keep the model near the observed temperatures appears consistent with observational studies of the mean heat flux. When measured in terms of the skill in simulating the Niño-3 SST, the NASA Coupled Climate Dynamics Group (CCDG) model and FSU wind products provide the highest skill.

1. Introduction

This paper presents tests of a numerical ocean model. The model has been developed as a component of a coupled ocean–atmosphere–land climate model for the purposes of studying interannual to interdecadal climate variability, a key element of which is the forecasting of the El Niño–Southern Oscillation (ENSO) phenomenon. As with any ocean model, off-line testing with prescribed atmospheric forcing is helpful in determining the suitability of the model for forecast work, but it is also fraught with difficulties since it is easy to build in the "correct" answer via the forcing. We measure the success of the simulation by its ability to recreate the sea surface temperature distributions when given the observed wind anomalies. The prescription of surface heat fluxes, however, is a much more difficult problem since good observations of these are not available. In fact, observations of the long-term annual mean of the surface heat flux are notoriously bad and can lead to serious errors in the simulated temperature distribution.

The model is an outgrowth of a two-layer upper ocean model with an explicit turbulent mixed layer (Schopf and Cane 1983) but with the addition of more layers. In experiments with the 2.5-layer model (Schopf and Suarez 1988), we had the problem that shallow layers interfaces would be brought up enough to surface, and it was necessary to mix water from the abyss into the active regions of the simulation. Thus there was a dependence on unmodeled portions of the system, which was disturbing. In order to overcome this, we needed to add more layers to extend down far enough to overcome surfacing of the abyssal layer.
Rather than adopt the modified sigma coordinate used by Gent and Cane (1988), we undertook the use of isopycnic vertical coordinates while maintaining the reduced-gravity approximation common to its equatorial upper-ocean forebears. For the purposes of simulating short-timescale processes such as El Niño and interannual variability, the use of reduced gravity models has been known to provide significant computational savings, and a layered model approach seems a natural way to attack such a problem. Replacement of this reduced gravity approximation and introduction of bottom topography would make this a full ocean circulation model.

The advantages of using isopycnic coordinates for the treatment of ocean dynamics have been understood for quite some time, and a number of models have been constructed using them (Bleck and Boudra 1981, 1986; Bleck et al. 1989; Bleck and Smith 1990; Oberhuber 1993). The chief advantages are the reduction of cross-isopycnic mixing by numerical diffusion and a reduction in the number of degrees of freedom needed to accurately represent the vertical modes. The first advantage helps the model maintain a sharper thermocline, the second helps speed up the calculation.

Weighing against this treatment are difficulties that isopycnic coordinate models have with 1) treating turbulent surface mixed layers, 2) handling the disappearance of layers where isopycnic surfaces intersect the surface or topography, and 3) maintaining similar vertical resolution throughout a global ocean. The first two problems are easily treated, and both Bleck et al. (1992) and Oberhuber (1993) have derived approaches similar to those used here. The third problem is more difficult and may not be treatable without the addition of more layers—thus overcoming the second advantage mentioned above.

In section 2 we present the model equations and formulation in isopycnic coordinates. Section 3 outlines the hindcast strategy—detailing the spinup procedure and anomaly driving employed for the hindcasts. It discusses the results of the spinup of a global version of the model forced with climatological conditions. These runs form the baseline climatology from which the hindcasts are made. Section 4 describes the sources for the wind anomalies used in the hindcast runs. Section 5 presents the results of the hindcast attempts in terms of the SST anomalies in the tropical Pacific and the depth of the 20°C isotherm.

2. Model equations

a. Vertical coordinate

The model is written in latitude φ—longitude λ orthogonal curvilinear coordinates with a generalized vertical coordinate ζ. The relationship between the generalized coordinate and z (distance from the earth’s reference geoid) is defined by the metric h as shown in Fig. 1:

\[ h = \frac{\partial z}{\partial \zeta}. \]  

In the finite-difference version of the code, it will be convenient to discretize the vertical domain into a set of layers, and so we choose ζ to increment by 1 between each layer interface. The average value of h between two-layer interfaces is therefore equal to the thickness of the layer. Since there is no need for h to be more than piecewise continuous, we shall define h to be constant between integral values of ζ.

b. Continuity

The mass continuity equation is

\[ \frac{\partial h}{\partial t} + \nabla \cdot (v h) + \frac{\partial w_r}{\partial \zeta} = 0, \]  

where \( w_r \) is the volume flux across ζ surfaces.

The mass flux across the surface of the ocean (ζ = 0) is due to evaporation and precipitation. Mass fluxes through lateral boundaries are zero.

c. Heat equation

The heat equation is

\[ \frac{\partial h T}{\partial t} + \nabla \cdot (v h T) + \frac{\partial w_r T}{\partial \zeta} = \frac{\partial}{\partial \zeta} \left( \kappa \frac{\partial T}{\partial h} \right) \]

\[ + \frac{\partial Q}{\partial \zeta} + h F_r(T), \]  

where \( Q \) is the external heat flux (latent and sensible and radiative fluxes at the surface, penetrative shortwave fluxes at depth). Heat flux \( Q \) is defined with positive fluxes upward; \( F_r \) is a horizontal smoothing operator (Shapiro 1970).

![Fig. 1. Vertical coordinate structure.](image-url)
d. Hydrostatics

In the vertical we employ a hydrostatic Boussinesq approximation. In $z$ coordinates
\[
\frac{\partial P}{\partial z} = -g \rho, \quad (4)
\]
while in $\zeta$ coordinates
\[
\frac{\partial P}{\partial \zeta} = -g \rho h. \quad (5)
\]

The major change in density is due to compressibility. We let $\rho$ be expressed as
\[
\rho = \rho_0 + \rho_1(z) + \rho^*(\lambda, \phi, z, t). \quad (6)
\]

This definition is made so that a water column with constant reference potential temperature $\Theta_d$ and salinity $S_d$ has $\rho^* = 0$ throughout.

The pressure at any depth is then given by
\[
P = \bar{P}(z) + P^*(\lambda, \phi, z, t), \quad (7)
\]
where
\[
\bar{P}(z) = -g \rho_0 z - g \int_{z_0}^{z} \rho_1 dz, \quad (8)
\]
\[
P^* = g \rho^*(0) \eta - g \int_{z_0}^{z} \rho^* dz, \quad (9)
\]
and $\eta$ is the height of the sea surface above the reference geoid.

e. Buoyancy

Since $\bar{P}$ does not enter in the pressure gradient force, we define buoyancy as
\[
b = -\frac{g \rho^*}{\rho_0}. \quad (10)
\]

The buoyancy of water at $(\Theta, S, p)$ is therefore given by
\[
b = -\frac{g[\rho(\Theta, S, p) - \rho(\Theta_d, S_d, p)]}{\rho_0} = -\frac{g \rho^*}{\rho_0}. \quad (11)
\]

The hydrostatic equation
\[
\frac{\partial P^*}{\partial z} = -g \rho^* \quad (12)
\]
can therefore be written
\[
\frac{\partial P^*}{\partial \zeta} = \rho_0 bh. \quad (13)
\]

In the reduced-gravity approximation we assume that horizontal gradients of $P^*$ vanish below the deepest interface. This assumption is equivalent to a definition of $\eta$ as
\[
\eta = \frac{1}{g} \int bhd\zeta. \quad (14)
\]

The model is formulated to treat both temperature and salinity, but for the initial experiments conducted here, with our focus on upper tropical ocean dynamics, we have simplified the equation of state by making buoyancy a simple function of temperature only. This process is akin to assuming a water mass relationship between temperature and salinity, substituting this in the equation of state and then truncating the resultant power series expansion. Rather than going through this three-step process with its empirical temperature–salinity $(T–S)$ relationship, we chose to proceed directly to an empirical fit of buoyancy to temperature. The input data were the surface temperature and salinity data of Levitus (1982), converted to density via the UNESCO equation of state (Millero and Poisson 1981). The least squares fit of buoyancy to temperature was weighted by the surface area.

Two special features of this fit are worth noting: First, since we intend to use the model in this study to examine the behavior in the tropical Pacific, we used data only from the Pacific basin in making this fit. There is a strong difference in the $T–S$ relationship between the North Atlantic and North Pacific that will influence any such simple fit. An extension of the model to other problems will have to use a different fit or the full equation of state.

Second, we only use surface values in determining the fit. The internal ocean distribution of buoyancy is largely determined by the wind forcing and the surface pattern of buoyancy through potential vorticity dynamics. The internal vertical modes, the pressure distribution, and the resultant expression of the surface forcing into the vertical distribution of currents are determined by the buoyancy profile. Typical layered reduced-gravity models assign a buoyancy to each layer and investigate the internal-mode dynamics. The fit of temperature to buoyancy is therefore needed primarily to map surface values of buoyancy to surface temperatures so that we may determine the heat flux and surface boundary condition.

The fit we use is cubic:
\[
b = b_0 + \alpha(T - T_r) + \alpha_2(T - T_r)^2 + \alpha_3(T - T_r)^3, \quad (15)
\]
where $b_0 = 1.68946 \times 10^{-3}$ m s$^{-2}$, $\alpha = 7.53775 \times 10^{-4}$ m s$^{-2}$ K$^{-1}$, $\alpha_2 = -4.02101 \times 10^{-5}$ m s$^{-2}$ K$^{-2}$, $\alpha_3 = 2.63299 \times 10^{-6}$ m s$^{-2}$ K$^{-3}$, and $T_r = 0^\circ$C. The rms difference between this function and the buoyancies computed from the full UNESCO equation of state are 0.0044 m s$^{-2}$. The rms variance of the input buoyancy data over the sampled domain is 0.0181 m s$^{-2}$.
f. Horizontal momentum equations

The horizontal component of the momentum is governed by
\[
\frac{\partial (\mathbf{v} h)}{\partial t} + \nabla \cdot (\mathbf{v} h \mathbf{v}) + \frac{\partial w \mathbf{v}}{\partial z} = -\frac{h}{\rho_0} \nabla P' - bh \nabla z - fhk \times \mathbf{v} + \frac{\partial}{\partial z} \left( \frac{\nu}{h} \frac{\partial \mathbf{v}}{\partial z} \right) + \frac{1}{\rho_0} \frac{\partial \tau}{\partial z} + h\mathbf{\Omega}_e \mathbf{v}.
\] (16)

The model equations in spherical geometry are presented in appendix A. For the complete finite-difference equations, see appendices B and C.

g. Vertical diffusion

Vertical mixing is parameterized through a Richardson number–dependent implicit vertical mixing scheme, with the additional treatment that turbulence at the ocean surface is modeled with a bulk turbulent energy mixed layer. The mixing is designed to handle three types of mixing: 1) internal shear-induced mixing, as is found in the equatorial upper thermocline near the strong shear regions of the equatorial undercurrent; 2) the surface turbulent mixing generated by wind stirring and surface cooling; and 3) convective overturning. These mixing schemes have been derived from the work of Kraus and Turner (1967) and Pacanowski and Philander (1981). The second is discussed in the next section.

The Richardson number–dependent portion of the mixing parameterization is based on the approximations of Pacanowski and Philander (1981), slightly modified for computational efficiency:
\[
\nu_r = \frac{\nu_0}{(1 + 5 \text{ Ri})^2} + \nu_b,
\] (17)
\[
\kappa_r = \frac{\kappa_0}{(1 + 5 \text{ Ri})^2} + \kappa_b,
\] (18)

where
\[
\text{Ri} = \frac{\partial b}{\partial z} \left( \frac{\partial v}{\partial z} \left( \frac{\partial v}{\partial z} \right)^{-1} \right).
\] (19)

h. Surface mixed layer

The topmost layer of the model is treated as a bulk turbulent well-mixed layer. All of the mixing effects are incorporated in the model through the prescription of a cross-coordinate mass flux at the base of the first layer. This entrainment is calculated through a balance of wind stirring, release of mean kinetic energy due to shear at the base of the layer, dissipation, and the increase in potential energy due to mixing, largely as set forth in Niiler and Kraus (1977). Here we shall discuss the changes from Niiler and Kraus (1977) as implemented in the model, principally for the solution of finite integration times and treatment of penetrating radiation.

In order to estimate the entrainment rate, we calculate the production rate of turbulent plus potential energy due to wind work and penetrating radiation \( P_r \). This change would occur in the absence of any turbulent deepening:
\[
P_r = mu^* + J_o(1 - e^{-\gamma h})\gamma^{-1}.
\] (20)

where \( J_o \) is that part of the surface buoyancy flux that penetrates into the water column. This is assumed to decay exponentially with a length scale of \( \gamma^{-1} \). The coefficient \( m \) is a constant, and \( \mu^* \) is the surface friction velocity. The exponential term is added here because in many equatorial cases \( h \), the mixed layer depth, will be significantly shallower than the e-folding length scale for penetrating radiation. Values for the mixing coefficients are given in Table 1.

Next, we compute the change in potential plus turbulent energy that is needed to homogenize the mixed layer after surface buoyancy fluxes and bulk dissipation within the mixed layer are added. As in (20), the effect is the same as discussed in Niiler and Kraus (1977), with the exception of the exponential term in the penetrating radiation effect. This value is proportional to the mixed layer depth times a quantity \( D \), given by
\[
D = \frac{1}{4} \left[ (1 + n)B_0 + (1 - n) |B_0| \right] + 2J_o(1 + e^{-\gamma h}) + \epsilon,
\] (21)

where \( B_0 \) is that part of the surface buoyancy flux that is absorbed at the surface, \( n \) parameterizes extra dissipation when the surface flux is negative, and \( \epsilon \) is a background dissipation rate.

<table>
<thead>
<tr>
<th>Table 1. Parameter and coefficient values.</th>
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<tbody>
<tr>
<td>Parameter</td>
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<tr>
<td>Vertical mixing</td>
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<td>( \nu_0 ) (m$^2$s$^{-1}$)</td>
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<td>( \nu_b ) (m$^2$s$^{-1}$)</td>
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<td>( \kappa_0 ) (m$^2$s$^{-1}$)</td>
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<td>( \kappa_b ) (m$^2$s$^{-1}$)</td>
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<tr>
<td>Mixed layer</td>
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<td>( n )</td>
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<tr>
<td>( \gamma ) (m$^{-1}$)</td>
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<td>( \epsilon )</td>
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<td>( q' ) (m$^2$s$^{-2}$)</td>
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<tr>
<td>( s )</td>
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<td>( m )</td>
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<td>Filtering</td>
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<td>Order of mass filter</td>
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<td>( \tau_f ) (day) for mass filter</td>
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<td>Order of ( T ) filter</td>
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<td>Order of ( v ) filter</td>
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<td>( \tau_f ) (day) for ( v ) filter</td>
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The third term of interest is the rate at which work must be done in order to deepen an already-mixed layer. As in Niiler and Kraus (1977),
\[
\frac{\partial E}{\partial h} = \frac{1}{2} \left( c_i^2 + q^2 - s \Delta u^2 \right) + D, \tag{22}
\]
where \( c_i^2 = h \Delta b \), \( \Delta b \) is the buoyancy jump at the base of the mixed layer and \( \Delta u^2 \) is the square of the magnitude of the shear at the base of the mixed layer. Here \( q^2 \) is the energy needed to agitate the entrained water. If \( \partial E/\partial h > 0 \), then work must be done to deepen the mixed layer.

Given the production (20), the rate at which work must be done to maintain the mixed layer at its current depth (\( hD \)), and the rate at which work must be done to deepen the mixed layer (22), we need to first decide whether the layer is to deepen or shallow. If \( P_r > hD \), then there is sufficient surface production to mix the surface buoyancy effects down through the existing layer and to deepen, and we arrive at an entrainment equation that is essentially that of Niiler and Kraus (1977):
\[
w_e = \frac{P_r - hD}{\partial E/\partial h}. \tag{23}
\]
If \( P_r < hD \), then it shallows to the Monin–Obukhov depth—the depth at which the surface production equals the change in potential energy by mixing:
\[
L^* = \frac{P_r}{D}. \tag{24}
\]
If \( \partial E/\partial h < 0 \) then deepening releases more turbulent energy than is needed to mix the mixed layer with the water beneath, and the algorithm is unstable. In the model, we only examine the shear between the mixed layer and the underlying coordinate layer. Any deepening that would occur would eventually be limited as the shear is used up and the mixed layer deepens through the coordinate layers. In this case, we limit the deepening at a large positive value. One remaining issue arises, however, if strong surface heating is occurring over strong shear (\( \partial E/\partial h < 0 \) and \( L^* > 0 \)). In this case, we assume that the layer goes to the Monin–Obukhov depth.

In the model code, entrainment is computed once per day. Where the mixed layer would deepen, an entrainment rate is stored, and the cross-coordinate mass flux at the base of the mixed layer is set to the entrainment rate. Where the mixed layer would shallow, the Monin–Obukhov depth is stored, and the cross-coordinate mass flux is computed by damping the mixed layer depth to the prescribed \( L^* \), with a decay time of 1 day.

1. Convection

When the water column becomes statically unstable, the mixing coefficients become very large, and the implementation of vertical mixing through an implicit algorithm in the model code allows for strong homogenization of the water column. The model algorithm separates such a vertical turbulent flux from any vertical advective fluxes, although the two may well become closely coupled through the dynamics of the system. At high latitudes, under conditions of surface cooling, the surface waters can become more dense than the abyss. In response, the vertical mixing will neutralize the water column without a net mass flux across the interfaces—we hypothesize strong upward and balancing downward motions on a subgrid scale. A secondary consequence of this neutralization, however, is that any convergent motion (as might be induced by wind forcing) is not opposed by pressure forces. Layers can become extremely thick, with a net downward motion. Imposition of limits on the layer thicknesses (see the following section) will result in a net downward mass flux across the deepest-layer interface, representing the sinking motion associated with bottom water formation. Thus convection and vertical motion are treated as related but distinct processes.

Appendix D discusses the algorithm used for ensuring that the water column becomes homogenized by this mixing. Appendix E discusses the numerics associated with having very large mixing coefficients.

j. Cross-coordinate mass fluxes

As was the case in Schopf and Cane (1983), the adoption of the generalized layered coordinate introduces the freedom to choose either \( h \) or \( w_e \), but not both. This freedom can be used in different ways throughout the domain and lies at the heart of the blending of a mixed layer model with an isopycnic model. We employ a four-step algorithm for determining the entrainment.

1) Assume no cross-coordinate mass fluxes and propagate the layer thicknesses to the new time level.
2) Set the entrainment at the bottom of the mixed layer according to the mixed layer equations.
3) Compute a provisional entrainment for the other layers designed so that the effect of \( w_e \partial b / \partial z \) is to maintain the interface at a prespecified isopycnic.
4) Impose limits on the new layer depths and thicknesses (i.e., modify the provisional values of \( w_e \) so that the layers stay within bounds).

When the value of \( w_e \) is computed, it is used for advection of mass, heat, and momentum. The first step leads to the provisional thicknesses and depths (\( h^* \) and \( z^* \)):
\[
h^*(t + \Delta t) = h(t - \Delta t) - 2 \Delta t \nabla \cdot (vh), \tag{25}
\]
\[
z^*(t + \Delta t) = \int_{-h}^{h^*(t + \Delta t)} d\zeta. \tag{26}
\]
In the second step, we examine the entrainment predicted by the mixed layer equation. If it is positive, we set \( w_e^* (\zeta = -1) \) by (23). If not, we set

\[
w_e^* = \frac{z^* - L^*}{\tau_e},
\]

where \( \tau_e \) is a time constant of 1 day.

For the third step, we note that with the centered differencing used in vertical advection, the numerical equivalent of \( w_e \partial b / \partial z \) is given by

\[
w_e \frac{\partial b}{\partial z} = w_e \left[ \frac{(b_k - b_{k+1})(h_k^* + h_{k+1}^*)}{2h_k^* h_{k+1}^*} \right].
\]

To make this damp the interface to a specified buoyancy \( b_{0} \), we want \( w_e \partial b / \partial z = (b_0 - b_0) / \Delta t \), where \( b_0 \) is the old interfacial buoyancy:

\[
b_0 = \frac{b_0 h_{k+1} + b_{k+1} h_k^*}{h_k^* + h_{k+1}^*}.
\]

We therefore arrive at a provisional cross-coordinate flux of

\[
w_e^* = \frac{2}{\Delta t} \frac{(b_i - b_0)}{(b_k - b_{k+1})} \frac{h_k^* h_{k+1}^*}{(h_k^* + h_{k+1}^*)}.
\]

This procedure is not exact, since advection across interfaces \( k - 1 \) and \( k + 2 \) will also influence the interfacial buoyancy. Furthermore, it cannot handle situations where \( b_i < b_{k+1} \) or \( b_i > b_{k} \) (which includes situations with unstable profiles). We first test for \( b_i < b_{k+1} \). If this occurs, the layer deepens with

\[
w_e^* = \frac{h_{k+1}^*}{2 \Delta t}.
\]

A second test is for \( b_i > b_{k} \). Under these conditions, the interface moves up with

\[
w_e^* = -\frac{h_k^*}{2 \Delta t}.
\]

After a first estimate of \( w_e^* \) is made for all layers, limit checking is done on the new layer thicknesses and depths. We compute

\[
z''(k) = z^*(k) + 2 \Delta t w_e^*(k)
\]

and then limit \( z'' \) so that

\[
z_{\min}(k) < z''(k) < z_{\max}(k)
\]

\[
z''(k - 1) + h_{\min}(k - 1) < z''(k) \quad < z''(k - 1) + h_{\max}(k).
\]

Finally, we recompute \( w_e \) as

\[
w_e = \frac{z''(k) - z(k, t - 2 \Delta t)}{2 \Delta t}.
\]

**k. Horizontal smoothing**

The horizontal smoothing operator is a modified Shapiro (1970) filter, applied to the mass, temperature, and momentum fields. The modifications include treatment of boundary conditions, application of a conservation correction, treatment in two dimensions, and partial application of the filtering effects.

The boundary conditions are treated in two ways. For mass and temperature, a full eighth-order filter is applied at all grid points. Lateral boundaries lie midway between grid points, and the boundary condition assumes that all odd derivatives vanish. For momentum, the boundaries lie at grid points, and it is assumed that all velocities vanish. Rather than invent higher-order boundary conditions for velocity, we simply reduce the order of the filter near a wall: at the first grid point in from the boundary, the result of a second-order filter is used to update the value; at the second, the result of a fourth-order filter is used, and so on until the interior of the domain is reached where the full order is applied. This technique does not require any information to be extrapolated into land.

The implementation of the filter involves repetitive applications of

\[
dX_{i+1/2}^{n+1} = X_{i+1}^{n} - X_i^{n},
\]

\[
X_{i+1}^{n+2} = 2X_{i+1}^{n+1} - X_{i}^{n+1},
\]

where \( X_i^{0} \) is the original value and \( X_i^{n/2} \) is the correction field to be applied. In order to conserve a weighted sum of \( X_i W_i \), we modify the final pass so that

\[
dX_{i+1/2}^{n+1} = \frac{(X_{i+1}^{n+1} - X_i^{n+1}) (W_{i+1} + W_i)}{2},
\]

and

\[
X_i^{n} = \frac{dX_{i+1/2}^{n+1} - dX_{i-1/2}^{n+1}}{W_i}.
\]

Such a conservative form is used for the mass field, with \( W = \cos \phi \).

In a two-dimensional problem, the simplest technique might be to apply the one-dimensional filter in the zonal direction, to be followed by another filter in the meridional. Because the boundary conditions are nonlinear, filtering in \( x \) first, then \( y \) does not produce the same answer as filtering in \( y \) first, then \( x \). To overcome this, a brute force method has been adopted that filters twice: the effects of the two-dimensional filter with \( x \) done first are averaged with the effects of the two-dimensional filter done with \( y \) first.

The filter is applied once per day. In a normal filter, the correction field is sufficient to entirely remove waves with wavelengths of \( 2 \Delta x \). If we let this correction field be denoted \( C \), then we apply \( \Delta t / \tau_f \), where \( \Delta t \) is the time interval between applications of the filter, and \( \tau_f \) is a timescale.
3. Experiment methodology and development of the model climatology

a. Overview

The hindcast of 1982–91 will be done by driving the model with different wind products. These wind products are the result of combining a standard wind climatology with wind anomaly estimates. In all of the runs, the wind climatology remains the same, only the anomalies change. In addition, the model needs a surface heat flux boundary condition. This can be constructed in a number of ways. A common approach is
to use a linearization of the bulk aerodynamical exchange equations as in Haney (1971). We reject this procedure because specification of a surface air temperature as observed for the 1982–91 period would "build in" the model results for simulation of the ENSO behavior. Instead, we will adopt an anomaly coupling scheme for the surface heat flux, in which the mean seasonal cycle of heat flux is prescribed, and surface temperature anomalies (such as El Niño) are weakly damped. We must therefore design a set of experiments to evaluate the mean surface heat flux cycle.

In general terms, the procedure is as follows.

1) Run the model with climatological winds, damp the SST to observed SST.
2) Rerun the model with climatological winds, damp the SST to an adjusted state. The purpose of this run is to bring the SST closer to observations in regions where the heat flux is large.
3) Rerun the model with climatological winds, and the mean cycle of heating from (2). This run should give approximately the same result as (2) and will be used as the "control" experiment.
4) Perform hindcast experiments with wind anomalies added to the climatological winds and heat fluxes specified as in (3). SST anomalies are defined as departures from the control run.

b. Initial damped solutions

The initial run is made with climatological monthly mean winds and a surface heat flux that damps the surface temperature to the observed monthly mean surface temperature climatology. The model is initialized from Levitus' climatology (Levitus 1982) and a geostrophic approximation for currents. It is then run for 20 years, and the last 10 years of this run are used for deriving mean climatologies of the model.
The wind fields used for this study were obtained from the Climate Analysis Center and are a blending of Hellerman and Rosenstein (1983) data with the newer estimates of Harrison (1989). Harrison’s data has improved computation of the drag coefficients but is not fully gridded and global. The blended product draws heavily from Harrison (1989) in the Tropics and from Hellerman and Rosenstein (1983) elsewhere.

The heat flux used in this run is evaluated on two grids: strong damping to the observed mean SST is done on a coarse (4° × 5°) grid, with a weaker damping of finer-scale features. This multiple-scale technique is used in order to accomplish two separate goals. First, we wish to diagnose a heat flux that would be required in order for the model to produce SST values close to climatology. We might want to make the damping so strong that we simply replace the model SST with climatological values each time step. Since we do not know what the true heat flux should be, this seems a valid way of estimating the flux. It conflicts, however, with a second objective: that the fine-scale eddy activities and fronts be damped realistically. By putting a strong restoring term on a coarse grid and weak one on the fine scale, we constrain the mean SST field to resemble the observations quite closely while still allowing eddy structures to be damped with moderate restoring coefficients.

Let the modeled sea surface temperature on the fine grid be given by \( T \) and the observed surface temperature on the fine grid be \( T_o \). The averages of these values

![Image of Fig. 5](image5.png)

**Fig. 5.** Difference between the annual mean surface temperature simulated in the SST adjustment run and that obtained from climatology.

![Image of Fig. 6](image6.png)

**Fig. 6.** Annual mean surface heat fluxes along the equator from (a) the initial run and (b) the SST adjustment run. Data are averaged from 4°S to 4°N.

![Image of Fig. 7](image7.png)

**Fig. 7.** Sea surface temperature averaged from 4°S to 4°N for 10 years of the control run. Contour interval is 1°C.
on the coarse grid will be denoted \([T]\) and \([T_o]\). We first construct the heat flux on the coarse grid as

\[
[Q] = K_o([T_o] - [T]), \tag{41}
\]

where \(K_o = -100 \text{ W m}^{-2} \text{ K}^{-1}\). Terms \([Q]\) and \([T]\) are then interpolated back to the fine grid. These values are \(\bar{Q}\) and \(\bar{T}\). On the fine grid, the heat flux is determined as

\[
\bar{Q} = \bar{Q} + K_1(\bar{T} - T), \tag{42}
\]

where \(K_1 = -25 \text{ W m}^{-2} \text{ K}^{-1}\).

With such a boundary condition, the large-scale features of the surface temperature are almost specified, and examination of these fields will tell us little about the quality of the model. Instead, the surface heat flux pattern is an independent measure. Figure 2 shows the annual mean surface temperature and surface heat flux from the initial spinup of the model. Figure 3 shows the difference between the annual mean SST and the observed SST.

c. SST adjustment

Since the heat flux parameterization used in the initial run is a simple damping to the observed SST, there will be a systematic bias in the simulated results, with the difference linearly proportional to the net heat flux. This bias can be partially corrected by replacing \([T_o]\) with

\[
[T_o^*] = [T_o] + \frac{f_o(\phi)[Q_i]}{K_o}, \tag{43}
\]

where \([Q_i]\) is the heat flux obtained in the first run. A latitudinal weighting factor \(f_o\) is introduced to restrict this adjustment to the Tropics. It is given by

\[
f_o = \frac{1}{2} \left[ \tanh \left( \frac{\phi + 30^\circ}{10^\circ} \right) \tanh \left( \frac{30^\circ - \phi}{10^\circ} \right) + 1 \right]. \tag{44}
\]

If the heat flux required by the ocean model is not sensitive to the small changes in SST necessary to bring
the model into agreement with the observations, this adjustment of the reference temperature should bring the simulated temperature into agreement with the observations. Term $[T^*_s]$ is evaluated for each month and saved on the $4^\circ \times 5^\circ$ grid.

The model run was continued for another 20 years, with the boundary condition that

$$[Q] = K_s([T^*_s] - [T]),$$

with $K_s$ retained at $-100 \text{ W m}^{-2}$.

Results for the last 10 years of this spinup are shown in Fig. 4a. The difference between the observed SST and the modeled temperature is shown in Fig. 5, and the annual mean of the heat flux along the equator for the initial run and the SST adjustment run are shown in Fig. 6.

d. Hindcast runs

During El Niño events, we do not expect that SST anomalies will be damped strongly to climatological values. As discussed in Schopf (1983), large-scale anomalies in SST induce large-scale changes in the atmosphere, which means that the damping is much weaker than would be predicted by a linearization of the heat flux parameterization about a fixed atmospheric state. In order to carry out the hindcast experiments, then, we need to modify the heat flux parameterization. We use

$$Q = \overline{Q}_c' + K_3(\overline{T}_c' - [T]), \quad (46)$$

where $\overline{Q}_c'$ and $\overline{T}_c'$ are the average annual cycles of heat flux and SST found in the SST adjustment run, and $K_3$ is a weak damping.

Fig. 11. Section of zonal current on the equator at 140°W. (a) From an average of the last 2 years of the control run. (b) From the TOGA current meter mooring data from 1983 to 1992. Regions with currents in excess of 1 m s$^{-1}$ are shaded.

Fig. 12. Section of zonal current on the equator at 110°W. (a) From an average of the last 2 years of the control run. (b) From the TOGA current meter mooring data from 1984 to 1991. Regions with currents in excess of 1 m s$^{-1}$ are shaded.
e. Control run

Because the hindcast runs are made with the same damping factor on the large and small scales, a hindcast run with no wind anomalies will not exactly reproduce the state found in the SST adjustment run. In the results that follow, most of the experiments were done with the damping factor equal to 10 W m\(^{-2}\) °C\(^{-1}\). We therefore have conducted a control experiment with this damping factor and no wind anomalies. The SST averaged from 4°S to 4°N for the 10-yr control experiment is shown in Fig. 7. A slight drift is seen over the first 3 years, followed by very stable repetitions of an annual cycle.

This control run serves to describe the model climatology in the absence of interannual wind anomalies. The annual mean surface height as defined by (14) is shown in Fig. 8. A difference of 60 cm is maintained across the Pacific, with a trough of about 20 cm at 10°N.

The annual mean temperature and undercurrent along the equator are shown in Fig. 9. For comparison, temperature data from the NMC ocean analysis system (Ji et al. 1994) is presented in Fig. 10. This product is the result of a data assimilation scheme that incorporates observed XBT and SST data into an ocean circulation model. The mean shown was constructed from analyses for the period from 1982 to 1993. In comparison with the ocean analysis, the control run of the present model maintains a reasonably sharp thermocline without any assimilation or restoration to subsurface data. Near 150°W, the separation between the 20° and 25°C isotherms is about 40 m in the simulation and about 50 m in the ocean analysis. In the western Pacific, there is a deep pool of water warmer than 28°C extending down to 90 m. In the east, near 110°W, the control run is warmer (by approximately 1°–2°C) than the ocean analysis from the surface down to 250 m.

The undercurrent reaches a maximum speed of slightly over 1 m s\(^{-1}\) at about 160°W, with its core at a depth of about 110 m. The undercurrent is quite flat, with the maximum at a depth of 100 m near 110°W. Since the ocean analysis product assimilates temperature, it is a good measure of the ocean state, but the undercurrent is not strongly constrained. A better measure of the currents comes from the TOGA current-
Fig. 14. Zonal wind stress anomalies (dyn cm\(^{-2}\)) averaged from 4\(^\circ\)S to 4\(^\circ\)N for four different regions across the Pacific.
4. Wind anomalies

The winds used to drive the hindcast experiments are derived from three sources:

1) an NMC global climate model run over observed sea surface temperatures for the period from 1982 through 1991 (MRF9);

2) a NASA Coupled Climate Dynamics Group (CCDG) global climate model run over observed sea surface temperatures for the period from 1980 through 1991 (CCDG);


With each of the datasets, the long-term record was analyzed to remove the mean annual cycle. We then reconstructed a full wind dataset by adding the analyzed anomalies to the Harrison–Hellerman–Rosenstein climatology. All wind data have been interpolated or averaged to a 4° latitude by 5° longitude grid and are monthly averages.

The NMC model is a T40 global spectral atmospheric GCM (Ji et al. 1994), which has been "tuned" for improving performance in the tropical Pacific, with particular attention paid to the behavior of the surface wind stress. Monthly sea surface temperature analyses were used for the lower boundary conditions, similar to the technique used in the Atmospheric Model Intercomparison Project (AMIP) (Gates 1992). No data assimilation was used in the atmosphere. Although the mean climatology of this model provides weak and somewhat unrealistic stresses on the equator, the stress anomalies may be sufficient to provide hindcast skill. The present hindcast run provides a rudimentary test of that skill, as will be shown below.

The CCDG model is a 4° × 5° gridpoint model. The model had been extensively tested and developed by running it with monthly climatological surface temperatures. To simulate the period from 1982 through 1991, the surface temperatures from AMIP were obtained and converted to monthly anomalies. These anomalies were then added to the surface temperature climatology that the CCDG model routinely uses. Thus, the run is not a true AMIP simulation, due to the small differences in the two climatologies. Again, no data assimilation was
done in the atmosphere, and the model was initialized by extending a run that had been conducted over climatological surface temperatures.

The FSU wind product is derived from the FSU pseudostress data, assuming the product of drag coefficient and air density to be $1.37 \times 10^{-3}$ kg m$^{-2}$. The monthly data were processed on a 2° grid from 30°S to 30°N, from 124°E to 70°W. Outside this region, the two products are blended at the edge of the FSU domain according to

$$
\tau = \tau_n + W(\tau_f - \tau_n),
$$

where $\tau_n$ and $\tau_f$ are the MRF9 and FSU stresses and

$$
W = \begin{cases} 
0, & x \leq 122.5^\circ \text{ or } x \geq 292.5^\circ \\
(x - 122.5^\circ)/15^\circ, & 122.5^\circ < x < 137.5^\circ \\
1, & 137.5^\circ < x < 277.5^\circ \\
(292.5^\circ - x)/15^\circ, & 122.5^\circ < x < 137.5^\circ \\
0, & y \leq -34^\circ \text{ or } y \geq 34^\circ \\
(y + 34^\circ)/12^\circ, & -34^\circ < y < -22^\circ \\
1, & -22^\circ < y < 22^\circ \\
(34^\circ - y)/12^\circ, & 22^\circ < y < 34^\circ.
\end{cases}
$$

All wind data are taken from monthly means, with no additional smoothing applied to the time series. The FSU data have more high-frequency variability than the others, but this is not found to have a large impact on the results.

---

**TABLE 2.** Root-mean-square variance $\sigma$ of the Niño-3 SST, correlation coefficient $\rho$, and scaled rms variance $S$ between the simulated and observed Niño-3 SST for the four wind experiments. The observed variance is 1.05.

<table>
<thead>
<tr>
<th>Wind</th>
<th>$\sigma$</th>
<th>$\rho$</th>
<th>$S$</th>
</tr>
</thead>
<tbody>
<tr>
<td>MRF9</td>
<td>0.59</td>
<td>0.65</td>
<td>0.76</td>
</tr>
<tr>
<td>CCDG</td>
<td>1.16</td>
<td>0.81</td>
<td>0.65</td>
</tr>
<tr>
<td>FSU</td>
<td>1.48</td>
<td>0.83</td>
<td>0.80</td>
</tr>
</tbody>
</table>
3) During 1986, the MRF9 winds have a westerly anomaly near the date line but easterly anomalies east of 150°W. The CDDG and FSU products show a slightly stronger westerly anomaly but significantly weaker opposing easterlies. They therefore show a significantly stronger zonal mean.

4) The CDDG and MRF9 winds show a dip in the westerly anomaly during March–May 1987 and have a stronger westerly anomaly in 1987 than in 1986. The FSU product maintains a more even westerly anomaly throughout the 1986–87 period.

5) During 1989, the FSU winds show significant easterly anomalies between 120°W and the date line, which persist throughout the year. The two models return to quite normal conditions, with only a small easterly anomaly between 160°W and 165°E.

6) During 1990, there appears to be westerly anomalies west of 170°W and easterlies to the east. The MRF9 and FSU products appear to have an even balance between these anomalies, while the CDDG winds enhance the westerly portion and have weaker easterlies. As a result, the CDDG product has an enhanced zonal mean westerly.

7) In the second half of 1991, the FSU product is showing strong westerlies across the basin. The two models have westerlies only in the west.

All of the wind anomaly products are added to the Harrison–Hellerman–Rosenstein wind climatology, gradually increasing the strength of the anomaly over the first six months. If $\bar{\tau}(t)$ represents the climatological stress cycle, and $\tau'(t)$ is the stress anomaly, the total stress used to drive the ocean model is

$$\tau = \bar{\tau}(t) + \frac{t - t_0}{t_1 - t_0} \tau', \quad t < t_1, \quad (49a)$$

$$= \bar{\tau}(t) + \tau', \quad t > t_1, \quad (49b)$$

where $t_0$ is 15 January 1982, and $t_1$ is 15 July 1982.

5. Hindcast results

a. Surface temperature

Each of the three wind anomaly products was used to simulate the period from 1982 through 1991. The first two products (MRF9) and (CDDG) are derived from climate models having no data assimilation or observational information other than the surface boundary conditions used throughout the experiments. For these experiments we chose $K_z = 10 \text{ W m}^{-2} \text{ K}^{-1}$. For comparison with the simulation results, Fig. 15a shows the observed surface temperature anomalies computed for the same period.

Figure 15b presents the result using the MRF9 winds, Fig. 15c shows the result using the CDDG winds, and Fig. 15d shows the SST computed using the FSU product. These figures present the difference be-
tween the hindcast experiment anomalies and the control run anomalies.

The observed behavior shows five basic events: the 1982–83 El Niño with anomalies up to 4°C persisting from mid-1982 through late 1983; a cool period from mid-1983 through mid-1986 with temperatures about 1.5°C cooler than normal; the 1987 El Niño of about 2°C; a cool period from early 1988 through early 1990 with anomalies of about 3°C; and a warm period starting in 1991.

Each of these periods is simulated in each of the models with varying degrees of success. The 1982–83 El Niño generally starts later in the simulations and seems to persist longer in the far eastern part of the basin. The late onset is perhaps due to the lack of a complete ocean initialization and the ramping-in of the wind anomalies. The MRF9 and CCGD wind products produce an early transition from warm to cold, while the FSU product causes the model to overestimate the warming—reaching 5°C in mid-1983.

All three wind products provide a cool period from late 1983 through mid-1986. The CCGD product gives the most coherent, strong cooling. Examining Fig. 14, we see that during this period, the MRF9 winds have a westerly anomaly in the eastern part of the basin that is not present in the FSU or CCGD product. This (and related differences in the meridional stress) is the likely source of the anomalously warm water along the coast that persists during 1984 through 1986. The variability in the FSU winds in the central and eastern part of the basin seems to lead to the breakup of the large-scale cooling pattern.

The transition to the 1987 El Niño is strongest in the FSU and CCGD runs. The dip in zonal mean westerly stress seen in the model products during early 1987 seems to lead to a weakening of the events in July that is not present in either the observations or the FSU-driven model. The MRF9 winds provide only a weak initiation of the event in early 1987, with the strongest warming in the last quarter of 1987, consistent with the low westerly stresses and opposing easterlies found during late 1986. The CCGD and FSU winds provide a sudden onset to the cold conditions in the beginning of 1988. The MRF9 winds have westerlies in the east at that time that appear to cause warming in the east and delay the appearance of cold water.

At the beginning of 1989, we noted the drop in easterly stresses in the MRF9 and CCGD products in the central basin. This terminates the cold condition rather suddenly at the start of 1989, in contrast with the observations and the model driven with FSU winds. During 1990, the MRF9 and FSU winds retain weak, almost neutral stresses across the basin, and the two models provide small cool conditions, while the observations seem to show a small amplitude in SST. In the second half of 1991, the FSU solution warms dramatically, consistent with the nearly uniform westerlies across the basin. The MRF9 model initiates a small warming, while the CCGD model allows the early warming to persist.

b. Niño-3 index

The Niño-3 SST is defined as the average surface temperature over the region from 5°S to 5°N, 90° to 150°W. Because of the way data are saved from the model, the nearly equivalent average from 4°S to 4°N, 87.5° to 152.5°W has been computed for the hindcast runs, as well as from the NMC analysis data. Figure 16 shows the Niño-3 SST anomalies for the MRF9, CCGD, and FSU experiments. Table 2 gives the rms variance of the time series, the correlation coefficient found between the simulated SST and that from the observations, and the scaled rms difference, as proposed by Busalacchi and Cane (1985). A scaled rms difference larger than unity indicates that the differences between the model and observation are greater than the variance of the observation.

The largest correlation coefficients (0.83) is found for the FSU wind anomaly run. The CCGD winds do almost as well, with a correlation coefficient of 0.81. The MRF9 winds have a smaller correlation at 0.65. Although the FSU correlation coefficient is largest, the run also has the highest rms difference between the result and observations. This indicates that while the winds provide good phasing of the events, the magnitude gives trouble. When the time series is split into two pentads, the FSU-driven model gives a high rms error for the period 1982–86 (0.96) and a low error for the last 5 years (0.56). This error is readily visible in Fig. 16. The CCGD winds provide the lowest rms scaled error (0.65) with a more even skill between the first and second pentads (0.57 and 0.63).

c. Depth of the 20°C isotherm

To examine the model's subsurface behavior, we have compared the results with the NMC ocean analysis product (Ji et al. 1994). While the ocean analysis is a model product, its constraint with observed XBT and SST data makes it a reasonable source for comparison for the 20°C isotherm (Z20).

Figure 17a shows the time-longitude behavior of the anomalies in the depth of the 20°C isotherm from the ocean analysis. Figure 17b–d show the corresponding behavior of the model simulations. The climatology is taken as the averages over the period from 1982 to 1991. As can be seen from a comparison of Fig. 9a and Fig. 10, the climatological mean for Z20 is approximately 15 m deeper than the ocean analysis in the western part of the basin and 20 m deeper in the east. There is not a large change in the mean values of Z20 between the control run and the anomaly experiments.

The MRF9-driven model provides the weakest excursions in Z20. The early termination of the 1982–83
El Niño in the MRF run is seen as clearly traceable to an early upwelling signal that propagates across the basin. This signal is seen to start in the CCDG and FSU and ocean analysis products as well but appears to die out near 150°W in January–March 1983. Examining the winds, we see strong easterly anomalies near 150°E in January 1983 in both models and the FSU analysis. The MRF9 model has a strong dip in the westerlies near the date line in the first quarter of 1983, and this is clearly reflected in the propagating feature seen in the Z20. The reduction of stress in the CCDG model in the center of the basin is weaker, and the shallowing signal is weaker but still present. In the FSU winds, there is no significant reduction of westerlies in the center of the basin, and the shallowing signal that starts from the far western part of the basin does not persist through the center of the basin.

In late 1983, all three models have a shallowing event propagating eastward. The FSU run provides the most shallowing near 150°W. This can be related to the strong easterly anomalies in the last quarter of 1983 found in the FSU winds.

The failure of the MRF9 run to start the 1987 El Niño properly can be seen as related to the large absence in a deepening signal that propagates from the west throughout 1986. The CCDG and FSU runs fail.
to simulate the deepening near the date line but pick up the strong event in the east at the start of 1987.

The anomalous shallowing of Z20 in the far western Pacific during 1987 is weakly simulated in the model runs. The CCDG and MRF9 runs provide a 10-m shallowing, with a peak of 20 m at the start of 1988, while the ocean analysis shows a maximum shallowing in the middle of 1987. The propagation to the east and the shallowing during early 1988 are captured in the FSU and CCDG runs. In the MRF9 run, westerly wind anomalies are found in the eastern part of the basin during early 1988, inhibiting the shallowing of Z20.

At the start of 1989, a deepening of 30 m in the west is seen in the ocean analysis product. This is captured in a somewhat reduced form in the FSU and CCDG runs, and a weak version is seen in the MRF9 run. In the MRF9 and CCDG runs, the signal appears able to propagate across the basin, appearing in mid-1989 as a strong deepening that is absent in the ocean analysis and FSU run. During this time, the FSU wind maintains a strong easterly stress in the middle of the basin that the two model products lack. This easterly stress is sufficient to keep the eastern part of the basin upwelling while depressing the isotherms in the west. When the models allow the easterly winds to relax, the signal gets through, and a premature warming ensues.

During early 1989, 1990, and 1991, there are three similar deepening events seen in the ocean analysis. They are simulated well in the FSU run, with only hints of the behavior in the others.
6. Discussion and conclusions

A reduced-gravity isopycnal ocean model has been developed for use in interannual to interdecadal climate studies. The model maintains a reasonably sharp thermocline and a strong equatorial undercurrent and requires moderate heat fluxes in order to recreate the observed annual cycle of sea surface temperature.

A methodology was developed so that the model could be run with wind anomalies only so that SST performance could be examined. In this methodology, a climatological cycle of heat flux was determined by spinup runs with the model forced with climatological winds and a strong damping to observed SST. In subsequent runs, this heat flux cycle is specified instead of the strong damping. This is the heat flux that is needed (with mean winds) to maintain the SST near observations. The hindcast was then run with a very weak damping back to climatological SST. The end result of this technique is that the model-predicted SST anomaly obtains no information as to the observed surface temperature.

Different representations of wind stress anomalies can create large differences in the model’s ability to simulate observed interannual surface temperature variations. At first glance, the three wind products look remarkably similar. Although the FSU product seems to have much more high-frequency variability, when smoothed in time it shows anomalies very similar to those of the models.

The testing developed here examines the sensitivity of the model SST to wind perturbations in the Tropics. Different choices of thermal damping were found to have a small impact on the model’s simulations, leading to the conclusion that wind stresses continue to be the major concern for hindcasting in the tropical Pacific. The statement can probably not be extended to higher latitudes or other domains, and even in the Tropics we cannot rule out important influences of the surface heat balance on SST.

None of the wind products seems to be clearly superior to the others. The FSU product is based on ship and buoy observations and can be expected to perform the best. The improvement after 1986 may be due to several factors, including the incorporation of TOGA TAO mooring data, increased ship reporting, and the decrease of spinup transients that may have occurred in the ocean model. The transition from no anomalies to full-strength anomalies that happens in the simulation of 1982 can have effects on ocean wave dynamics that may cause errors in the simulation of SST. While we feel that this is a more significant problem for the initialization of coupled models, it is hard to eliminate this feature from consideration.

The CCDG model-derived winds perform almost as well as the observed winds. Particular difficulties in the model-driven solutions seem to be the period from 1989 through 1991. In this period the model-derived winds generate excessive early warming in the Niño-3 SST, quite different from observations. During this time, the FSU winds provide a very good simulation of the Niño-3 SST. This is the period when the winds in the eastern Pacific are most noticeably different between the FSU and model-derived products. During 1989, strong easterly anomalies in the eastern Pacific probably drive increased upwelling and keep the ocean model from generating the warm anomalies found in the CCDG and MRF9 runs. By the end of 1991, however, the FSU winds show a strong westerly anomaly in both the central and eastern part of the Pacific. These lead to a strong warm anomaly in 1991—a year that produced early indications of an El Niño event but that never developed to full strength. The CCDG and MRF9 winds provide a better description of the 1991 behavior—both have easterly anomalies in the eastern Pacific.

In recent coupled GCM experiments (Mechoso et al. 1995), a wide variety of behaviors is found with different models and models run with slightly different parameterizations. As mentioned in Mechoso et al. (1995), some models produce very weak interannual variability, while others seem able to capture some (if not all) of the variability found in nature. The experiments shed some light on the high degree of sensitivity of coupled models and wide differences in their abilities to simulate El Niño events. The MRF9 winds appear at first glance to be very similar to the CCDG winds, but the slight difference in behavior—particularly the tendency of the MRF9 model to put opposing anomalies east and west of about 140°W—leads to dramatic changes in the ocean model’s SST response. A good deal of work has been done by modeling groups to tune the surface stresses to give reasonable magnitudes in the zonal stress anomalies for a given SST anomaly. These results may well indicate that further attention needs to be paid to the structure of these anomalies as well as the magnitude.

The hindcast experiments were designed to document the response of the ocean model to the types of wind fields likely to arise from atmospheric GCMs and to evaluate the suitability of the model for coupled simulations. As such a test, these runs demonstrate a first-order verification that the model can produce SST anomalies similar to those observed in nature under conditions that might be expected to arise in a coupled model simulation. The fact that SST anomalies of the right magnitude are produced when the model is driven by the CCDG winds implies that the two models are likely candidates for coupled simulations. Use of these two models has therefore been made for coupled GCM-based forecast experiments (Mechoso et al. 1995).

Acknowledgments. NMC wind products and ocean analyses were provided by Drs. Ming Ji and Ants
Leetmaa of NOAA/NMC. The TOGA current meter mooring data was obtained from the TOGA TAO Project Office, Dr. Michael J. McPhaden, director. This work was supported by NASA RTOP 578-41-45-78 and the NOAA TOGA Program on Prediction (T-POP). Model documentation and source code is available via World Wide Web at http://ccdg.gsfc.nasa.gov.

APPENDIX A

Equations in Spherical Coordinates

The model equations in component form are

\[
\frac{\partial h}{\partial t} + \frac{1}{a \cos \phi} \frac{\partial (uh)}{\partial \lambda} + \frac{1}{a \cos \phi} \frac{\partial (uh \cos \phi)}{\partial \phi} + \frac{\partial w_r}{\partial \zeta} = 0 \tag{A1}
\]

\[
\frac{\partial (\Theta h)}{\partial t} + \frac{1}{a \cos \phi} \frac{\partial (uh \Theta)}{\partial \lambda} + \frac{1}{a \cos \phi} \frac{\partial (uh \Theta \cos \phi)}{\partial \phi} + \frac{\partial (w_r \Theta)}{\partial \zeta} = \frac{\partial}{\partial \zeta} \left( \kappa_0 \rho^2 \frac{\partial \Theta}{\partial \zeta} \right) + \frac{1}{\rho \partial} \frac{\partial Q_\Theta}{\partial \zeta} + h q'(\Theta) \tag{A2}
\]

\[
\frac{\partial (uh)}{\partial t} + \frac{1}{a \cos \phi} \frac{\partial (uh u)}{\partial \lambda} + \frac{1}{a \cos \phi} \frac{\partial (uh v)}{\partial \phi} + \frac{\partial (w_r u)}{\partial \zeta}
\]

\[
= - \frac{h}{\rho} \frac{\partial P'}{\partial \lambda} - \frac{bh}{a \cos \phi} \frac{\partial z}{\partial \phi} + \left( f + \frac{u \tan \phi}{a} \right) u_h + \frac{\partial}{\partial \zeta} \left( \nu \rho^2 \frac{\partial u}{\partial \zeta} \right) + \frac{1}{\rho \partial} \frac{\partial \tau^\lambda}{\partial \zeta} + h q'(u) \tag{A3}
\]

\[
\frac{\partial (uh)}{\partial t} + \frac{1}{a \cos \phi} \frac{\partial (uh v)}{\partial \lambda} + \frac{1}{a \cos \phi} \frac{\partial (uh v)}{\partial \phi} + \frac{\partial (w_r v)}{\partial \zeta}
\]

\[
= - \frac{h}{\rho_0} \frac{\partial P'}{\partial \phi} - \frac{bh}{a \cos \phi} \frac{\partial z}{\partial \phi} - \left( f + \frac{u \tan \phi}{a} \right) u_h + \frac{\partial}{\partial \zeta} \left( \nu \rho^2 \frac{\partial v}{\partial \zeta} \right) + \frac{1}{\rho \partial} \frac{\partial \tau^\phi}{\partial \zeta} + h q'(v) \tag{A4}
\]

\[
\frac{\partial P}{\partial \zeta} = \rho_0 h \tag{A5}
\]

\[
\frac{\partial P'}{\partial \zeta} = \rho_0 bh \tag{A6}
\]

\[
P'(0) = g \rho_0 \eta \tag{A7}
\]

\[
\eta = \frac{1}{g} \int_{\zeta}^0 bhd\zeta. \tag{A8}
\]

APPENDIX B

Model Difference Equations

\textbf{a. Grid}

The grid is a latitude–longitude rectangular array, with the origin of the coordinate system at \((\lambda_0, \phi_0)\). This is the southwest corner of the first grid box. Each grid box has an index in the zonal direction \((i)\) and the meridional direction \((j)\) and a spacing of \(\Delta \lambda\) by \(\Delta \phi\). The grid box \(B_{ij}\) is bounded by

\[
\lambda_0 + (i - 1) \Delta \lambda < \lambda < \lambda_0 + i \Delta \lambda
\]

and

\[
\phi_0 + (j - 1) \Delta \phi < \phi < \phi_0 + j \Delta \phi.
\]

\textbf{b. Operators}

The following difference operators are defined as:

\[
\delta_{\lambda} X(\lambda) = X\left(\lambda + \frac{\Delta \lambda}{2}\right) - X\left(\lambda - \frac{\Delta \lambda}{2}\right) \tag{B1}
\]

In the vertical, layers are indexed by \(k\), where

\[
k - 1 < \zeta < k
\]

defines the limits of the layer. The layer indices range from 1 to \(KM\), with \(\zeta = KM\).

The grid-box properties \(h\) and \(T\) are defined at the center of the grid box, and the velocity \((u, v)\) is defined at the northeast corner. The entrainment velocity \(w_r\) is defined at \(\zeta = k\), beneath \(h\).
\[ \delta_y Y(\phi) = Y\left(\phi + \frac{\Delta \phi}{2}\right) - Y\left(\phi - \frac{\Delta \phi}{2}\right) \]  \hspace{1cm} (B2)

\[ \delta_z Z(\zeta) = Z\left(\zeta + \frac{1}{2}\right) - Z\left(\zeta - \frac{1}{2}\right) \]  \hspace{1cm} (B3)

\[ \bar{X}(\lambda) = \frac{1}{2} \left[ X\left(\lambda + \frac{\Delta \lambda}{2}\right) + X\left(\lambda - \frac{\Delta \lambda}{2}\right) \right] \]  \hspace{1cm} (B4)

\[ \bar{Y}(\phi) = \frac{1}{2} \left[ Y\left(\phi + \frac{\Delta \phi}{2}\right) + Y\left(\phi - \frac{\Delta \phi}{2}\right) \right] \]  \hspace{1cm} (B5)

\[ \bar{Z}(\zeta) = \frac{1}{2} \left[ Z\left(\zeta + \frac{1}{2}\right) + Z\left(\zeta - \frac{1}{2}\right) \right] \]  \hspace{1cm} (B6)

c. Equations

1) CONTINUITY

\[ \frac{\partial h}{\partial t} = -\frac{1}{a\Delta \lambda \cos \phi} \delta_y(\tilde{h} \tilde{u}^\phi) - \frac{1}{a\Delta \phi \cos \phi} \delta_x(\tilde{h} \tilde{u}^\phi) - \delta_z w_e. \]  \hspace{1cm} (B7)

2) HEAT

\[ \frac{\partial h^\phi}{\partial t} = -\frac{1}{a\Delta \lambda \cos \phi} \delta_y(h \tilde{u}^\phi) \cdot \delta_x(\tilde{h} \tilde{u}^\phi) - \delta_z w_e. \]  \hspace{1cm} (B8)

3) MOMENTUM

In order to construct the momentum equations, we need to define a few auxiliary terms. The first is the interpolated grid thickness at the grid-box corners. This is done with area weighting:

\[ h_u = \tilde{h} \frac{\hat{\lambda}}{\cos \phi} \]  \hspace{1cm} (B9)

\[ w_u = \tilde{w} \frac{\hat{\lambda}}{\cos \phi} \]  \hspace{1cm} (B10)

The next step is to define advective mass fluxes at the midpoints of the box walls:

\[ U^* = \tilde{u} \frac{\hat{\lambda}}{\cos \phi} \]  \hspace{1cm} (B11)

\[ V^* = \tilde{v} \frac{\hat{\lambda}}{\cos \phi} \]  \hspace{1cm} (B12)

So we write

\[ \frac{\partial (h_u u)}{\partial t} = -\frac{1}{a\Delta \lambda \cos \phi} \delta_y(U^* \tilde{u}) - \frac{1}{a\Delta \phi \cos \phi} \delta_x(V^* \tilde{u}) - \delta_z(w_u \tilde{u}^\zeta) \]

\[ \frac{\partial (h_v v)}{\partial t} = -\frac{1}{a\Delta \lambda \cos \phi} \delta_y(U^* \tilde{v}) - \frac{1}{a\Delta \phi \cos \phi} \delta_x(V^* \tilde{v}) - \delta_z(w_v \tilde{v}^\xi) \]

4) HYDROSTATICS

At the center of layer \( k \), the pressure is given by

\[ \frac{P'}{\rho_0} = \frac{b h_k}{2} + \sum_{l=k+1}^{k+M} b h_l. \]  \hspace{1cm} (B15)

APPENDIX C

Time Differentiating

The equations are time-split, with the hydrodynamics done with short explicit time steps, and the vertical diffusion, convective adjustment, and filtering done with much coarser time resolution. The hydrodynamics portion of the solution is defined as those terms solving for zonal, meridional, and vertical advection; Coriolis and metric terms; the pressure term; and the external forcing term. The overall evolution of the state variables (which we generically represent as \( X \)) is given by an equation of the form:

\[ \frac{\partial X}{\partial t} = \mathcal{H} + \mathcal{D} + \mathcal{F}, \]  \hspace{1cm} (C1)

where \( \mathcal{H} \) represents the hydrodynamics, \( \mathcal{D} \) the vertical mixing, and \( \mathcal{F} \) the horizontal filtering.
The hydrodynamics portion of the code is differenced with leapfrog time steps with an Asselin (1972) time filter. The leapfrog procedure is defined as

\[ X(t + \Delta t) = X(t - \Delta t) + 2\Delta t \mathcal{R}(X), \quad (C2) \]

where \( \Delta t \) is the time increment.

Time filtering is done by resetting the values at \( t \) to be

\[ X(t) = X(t) + a[X(t - \Delta t) - 2X(t) + X(t + \Delta t)], \quad (C3) \]

where \( a \) is the time-filtering factor (0.05 is used in the default model).

**APPENDIX D**

**Vertical Diffusion**

The calculation of the effects of vertical diffusion is typically done once per day, although a more frequent invocation might be better. The vertical diffusion routine handles both the effects of stable mixing and convective overturning through an iterative, semi-implicit technique.

The vertical diffusion routine simulates the effects of mixing over a finite-time interval \( \delta t \). It does this by splitting the time interval into a number \( n \) of subintervals and repeating the algorithm for each subinterval. The diffusion coefficients are computed at the start of each iteration, then an implicit time step is taken to advance the state. This means that the diffusion coefficients change during the iteration as the density and velocity field changes.

When the layers become statically unstable, the mixing coefficients are set to a high value \( (\kappa_c, \nu_c) \), and a flag is set to indicate that mixing has occurred. The convection criteria is set by testing the Richardson number. If it falls below a critical value \( [\mathcal{R} = O(10^{-3})] \), the mixing flag is set. Figure D1 shows a conditionally unstable profile and the mixing processes that would occur during an iterative scheme. In (a), the initial profile is shown, and the mixing coefficient at the interface marked UNSTABLE would be set to the high convective limit. The other layers would have the small background diffusivities associated with their stable interfaces. On the next iteration, (b) the originally unstable interface has become neutral, and a new unstable interface has shown up. If we compute the coefficients according to our simple algorithm, we will set the high mixing to the base of the third layer, and the mixing will set the profile to something like that shown in (c), where we see that the second interface has become unstable. Instead, we implement the scheme so that if the "mix" flag is true, the mixing coefficient is set to the convective value regardless of the present value of the stability. In this case the diffusivity at both the second and third layers would remain high, and the implicit step would bring the layer profile to that shown in (d). Any conditionally unstable profile can be set to fully stable with such a procedure as long as the number of iterations is equal to the number of layers and the mixing coefficients for the convective limit are large enough to fully mix the column.

The Richardson numbers and diffusivities are determined at the layer interfaces, immediately below the centers of the grid boxes. The viscosities are needed below the box corners in order to diffuse momentum. The model first computes the viscosities at the same points as the diffusivities, then averages the viscosity to the box corners. This enables us to use the convective testing to set high values for viscosity when unstable in a fashion similar to that done for diffusivity.

**APPENDIX E**

**An Algorithm for Implicit Mixing**

An implicit scheme for vertical diffusion in numerical models is often used when either diffusivities or layer thicknesses vary widely and can be used to replace vertical convective adjustment by allowing large values for diffusivities—well beyond those that would be permissible with explicit schemes. The nature of the scheme is to generate a tridiagonal matrix problem, which is usually solved with familiar techniques, such as that proposed in Richtmyer and Morton (1967).
In the attempt to use Richardson number–dependent mixing, and/or to replace convective adjustment, extremely large diffusivities can be generated, which leads to inaccuracies in numerical treatments, even with 128-bit arithmetic on a Cray C90. A simple revision of the algorithm, however, can be proposed that provides much greater accuracy, in which a 32-bit machine can provide better results.

The target equation to be solved is

$$\frac{\partial X}{\partial t} = \frac{\partial}{\partial z} \left( \kappa \frac{\partial X}{\partial z} \right).$$

(E1)

Its finite-difference version is

$$X^n_k - X^k_{k-1} = \frac{\Delta t}{h_k} \left( 2\kappa_{k+1/2} \frac{X^n_{k+1/2} - X^n_{k-1/2}}{h_{k+1/2} + h_k} - 2\kappa_{k-1/2} \frac{X^n_{k+1/2} - X^n_{k-1/2}}{h_{k-1/2} + h_k} \right).$$

(E2)

This is often written in the form

$$-A_k X^n_{k+1} + B^n_k X^n_k - C_k X^n_{k-1} = D_k,$$

(E3)

where

$$A_k = \frac{2\kappa_{k+1/2}\Delta t}{h_{k+1/2} + h_k},$$

(E4)

$$B_k = h_k + A_k + C_k,$$

(E5)

$$C_k = \frac{2\kappa_{k-1/2}\Delta t}{h_{k-1/2} + h_k},$$

(E6)

$$D_k = h_k X^n_{k-1},$$

(E7)

and then solved using

$$X^n_k = E_k X^n_{k+1} + F_k,$$

(E8)

$$E_k = \frac{A_k}{B_k - C_k E_{k-1}},$$

(E9)

$$F_k = \frac{D_k + C_k F_{k-1}}{B_k - C_k E_{k-1}}.$$

(E10)

Now it has been shown that as long as \( B > A + C \), that \( E_k < 1 \), and that \( F_k \) must remain “reasonable”, (Richtmyer and Morton 1967). For all cases that we are interested in, in which \( h \) and \( \kappa \) are positive definite, this requirement on \( B \) is satisfied, since all coefficients are greater than zero. But this statement was proven without regard for accuracy.

Particularly, note that \( C_k = A_{k-1} \), so we can write

$$E_k = \frac{A_k}{B_k - A_{k-1} E_{k-1}}$$

(E11)

$$= \frac{A_k}{h_k + A_k + A_{k-1}(1 - E_{k-1})}$$

(E12)

$$F_k = \frac{D_k + A_{k-1} F_{k-1}}{B_k - A_{k-1} E_{k-1}}$$

(E13)

$$= \frac{D_k + A_{k-1} F_{k-1}}{h_k + A_k + A_{k-1}(1 - E_{k-1})}.$$  

(E14)

The term of interest is the last one in the denominator \( A_{k-1}(1 - E_{k-1}) \). When \( A \gg h \), then \( E \approx 1 \). (Here \( E = 1 \) implies that an interface with high mixing coefficient will set the two adjacent layers to have the same value, a very good approximation.) The problem comes from trying to evaluate the denominator for the next layer, since \( (1 - E_k) \) vanishes while \( A_k \) goes to infinity. Suppose \( h \) and \( A \) are order 1 for all layers except \( k \), where \( A \) is order \( 1/e \). In this case,

$$E_k = 1 - O(e).$$

Suppose that the machine truncates the difference. Then the denominator for the next row will become \( (B_k + A_k)^{-1} \).

If we define \( \alpha_k = A_k(1 - E_k) \) and solve analytically, we find

$$\alpha_k = A_k(1 - E_k)$$

(E15)

$$= \frac{A_k [h_k + A_{k-1}(1 - E_{k-1})]}{h_k + A_k + A_{k-1}(1 - E_{k-1})}$$

(E16)

$$= \frac{A_k (h_k + \alpha_{k-1})}{h_k + A_k + \alpha_{k-1}}.$$  

(E17)

Note that \( \alpha_k \) is bounded by \( h + \alpha_{k-1} \) for large \( A \). Since \( \alpha_0 = 0 \), this means that \( \alpha \) is bounded by the depth of the interface, which is the same order as the other terms in the denominator.

The revised algorithm is therefore

$$\alpha_k = \frac{A_k (h_k + \alpha_{k-1})}{h_k + A_k + \alpha_{k-1}}.$$  

(E18)

$$E_k = \frac{A_k}{h_k + A_k + \alpha_{k-1}}.$$  

(E19)

$$F_k = \frac{D_k + A_{k-1} F_{k-1}}{h_k + A_k + \alpha_{k-1}}.$$  

(E20)

Example case. Suppose we have a case where

$$A_1 = 0$$  

(E21)

$$A_2 = 10^{10}$$  

(E22)

$$A_3 = 0$$  

(E23)

$$h_1 = 1$$  

(E24)

$$h_2 = 1$$  

(E25)

$$h_3 = 1,$$  

(E26)
then
\[
E_1 = \frac{10^{10}}{1 + 10^{10}}
\]
\[
\alpha_1 = \frac{10^{10}}{1 + 10^{10}}.
\]

On a 32-bit machine using IEEE arithmetic using the original algorithm, \(E_2\) evaluates to 1 and performing the multiplication \(A_2(1 - E_2)\) yields 0.0, not 1. Double precision (64 bit) is required to bring the result to within eight places of accuracy. When \(A_2\) is \(10^{20}\), double precision fails [giving a zero result for \(A(1 - E)\)]. On a 32-bit machine, it was possible to have \(A_2\) take on values up to \(10^{35}\) before encountering overflow when using the modified algorithm.

While the traditional methods for inverting the tridiagonal equations appear to be sufficiently accurate for cases with a range in diffusivities over 5 orders of magnitude (or 10 orders on a 64-bit machine), the new technique provides much higher accuracy and stability while only requiring one more multiplication operation per layer.

REFERENCES


